# *Suggested Solutions to:*
# Final Exam, Spring 2014
# Industrial Organization
# May 30, 2014

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## Question 1: Quantity setting, product differentiation, and strategic delegation

*To the external examiner: The students had not seen this problem before. But they have studied a problem set question about strategic delegation (with only two possible delegation actions: profit or revenue maximization) and we have in the course talked extensively about strategic incentives.*

### Part (a)

We can solve the game by backward induction, thus starting with the managers' decisions at stage 2. Manager $i$'s problem at stage 2 of the game can be written as:

$$\max_{q_i} (a - c + x_i - q_i - dq_j)\, q_i.$$

The first-order condition is:

$$-q_i + (a - c + x_i - q_i - dq_j) = 0.$$

Solving the two first-order conditions for $q_1$ and $q_2$ yields

$$q_i^* (x_1, x_2) = \frac{(2-d)(a-c) + 2x_i - dx_j}{4 - d^2}.$$

Note in particular that with $x_1 = x_2 = x$ we have

$$q^* (x) = q_i^* (x_1, x_2) \mid_{x_1 = x_2 = x} = \frac{a - c + x}{2 + d}.$$

We now turn to the owners' problem at stage 1. Owner $i$ maximizes the own firm's profits, which can be written as

$$\begin{aligned}
&\pi_i (q_1^* (x_1, x_2), q_2^* (x_1, x_2)) \\
&= V_i (x_i, q_1^* (x_1, x_2), q_2^* (x_1, x_2)) - x_i q_i^* (x_1, x_2) \\
&\qquad = [q_i^* (x_1, x_2)]^2 - x_i q_i^* (x_1, x_2),
\end{aligned}$$

with respect to $x_i$. The first-order condition is:

$$2q_i^* (x_1, x_2) \frac{\partial q_i^* (x_1, x_2)}{\partial x_i} - q_i^* (x_1, x_2) - x_i \frac{\partial q_i^* (x_1, x_2)}{\partial x_i} = 0.$$

Solving the two first-order conditions yields

$$x_1^* = x_2^* = x^* = \frac{(a-c)\, d^2}{4 + 2d - d^2}.$$

### Part (b)

The right answer is (iv).

*Explanations (not asked for in the exam and should not be provided by the student):*

- If $d < 0$ it is clear from the utility function stated in the question that $\frac{\partial^2 U}{\partial q_1 \partial q_2} = -d > 0$. That is, the consumer's marginal utility of consuming good 1 is increasing in the consumption of the other good, which means that the goods are complements.
- Moreover, if $d < 0$ , then a firm's best reply is increasing in the other firm's quantity (one can see, from the first-order condition above, that the slope of the best reply equals $-d/2 > 0$). This means that the firms' choice variables are strategic complements.

### Part (c)

- If $d = 0$, then demand for good 1 is independent of the demand for good 2, and each of the two firms is a monopolist. Since the two forms do not compete with each other, there is nothing an owner can gain strategically by instructing the own manager to maximize anything else than the own profits. Therefore, in this case we should expect $x = 0$.

- If $d > 0$, then the consumers perceive the goods as substitutes: if consuming more of one good, consumption of the other good becomes less attractive. In this sense the firms compete with each other and, indeed, a firm's profits will be larger if the rival firm lowers its output. By committing to some appropriate value of $x_i$, an owner can try to create an incentive for the rival firm to indeed lower its output. Should we expect this value of $x_i$ to be positive or negative? Note that, given $d > 0$, the choice variables of the managers are strategic substitutes. This means that an owner can create an incentive for the rival manager to lower its output by making the own manager more aggressive (in the sense of wanting to produce a lot). The way in which the owner can make the own manager more aggressive is to give him or her an instruction to care about output more than what is motivated by profit maximization — this amounts to making $x_i$ positive.

- If $d < 0$, then the choice variables of the managers are strategic complements. It also means, however, that the two goods are complements in the eyes of the consumers and because of this the firms would, at an equilibrium with $x_1 = x_2 = 0$, choose *too small* quantities (relative to the quantities that maximize joint profits). Firm 1 (say) would thus benefit if the other firm produced *more*. The way in which firm 1 can make firm 2 produce more, given strategic complements, is again to instruct the own manager to be more aggressive — which amounts to making $x_i$ positive.

### Part (d)

The other approach is called the **unit demand approach**. Here is an explanation of the idea (taken from the lecture slides):

- Assume that consumers (i) have unit demand (buy one unit of the good or nothing), (ii) have different tastes and (iii) form a continuum.
  - Example: the Hotelling model (to be studied later). See a sketch on next slide.

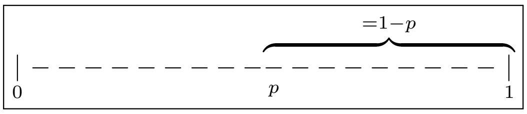
- There are a continuum of consumers with total mass one.
  - The net utility of a consumer:

$$\boxed{\begin{cases} r - p & \text{if buying one unit of the good} \\ 0 & \text{if not buying the good,} \end{cases}}$$

    where $p$ is the price and $r \in [0, 1]$ is a taste parameter.
  - Different consumers have different values $r$.
  - The distribution of $r$ values is **uniform** on $[0, 1]$.

- A consumer buys the good if and only if $\boxed{r - p \geq 0 \Leftrightarrow r \geq p}$.
  - So the mass of consumers who buy, given the price $p$, equals

$$\boxed{q = 1 - p.}$$




## Question 2: Collusion in a Cournot oligopoly with a fixed production cost

*To the external examiner: This question is identical to (parts of) a question in a problem set that the students discussed in an exercise class.*

### Part (a)

We must investigate under what conditions each one of the firms does not have an incentive to deviate from the strategy. In qualitative terms, there are three different situations we need to consider: (i) on the equilibrium path, the firm that is supposed to choose $q_{i,t} = 0$ must not have an incentive to deviate; (ii) on the equilibrium path, the firm that is supposed to choose $q_{i,t} = 6$ must not have an incentive to deviate; (iii) off the equilibrium path, neither firm must have an incentive to deviate from $q_{i,t} = 4$.

In situation (iii) it is clear that no firm would have an incentive to deviate, simply because

$(q_{1,t}, q_{2,t}) = (4,4)$ is a Nash equilibrium of the one-shot game. If expecting the other firm (say firm $j$) to choose $q_{j,t} = 4$, then the action that maximizes the current-period profits is indeed $q_{i,t} = 4$ (and the rival's actions in future periods will not change if deviating from $q_{i,t} = 4$).

Now consider situation (i): the incentives to deviate for a firm that is supposed to produce nothing. The present-discounted stream of profits for this firm, at the point when it is supposed to choose $q_{i,t} = 0$, equals

$$\begin{aligned} V^{eq} &= 0 + \delta\pi^m + 0 + \delta^3\pi^m + 0 + \delta^5\pi^m + \cdots \\ &= \delta\pi^m \left(1 + \delta^2 + \delta^4 + \delta^6 + \cdots\right) = \frac{\delta\pi^m}{1-\delta^2}, \end{aligned}$$

where $\pi^m$ denotes the single-period monopoly profits:

$$\pi^m = (12 - q_1^*)\, q_1^* - k = (12-6)\,6 - 8 = 28.$$

If deviating, the present-discounted stream of profits for this firm, at the point when it is supposed to choose $q_{i,t} = 0$, equals

$$\begin{aligned} V^{dev} &= \pi^d + \delta\pi^n + \delta^2\pi^n + \delta^3\pi^n + \cdots \\ &= \pi^d + \delta\pi^n \left(1 + \delta + \delta^2 + \delta^3 + \cdots\right) \\ &= \pi^d + \frac{\delta\pi^n}{1-\delta}, \end{aligned}$$

where $\pi^d$ denotes the best possible deviation profit if the other firm produces $q_{j,t} = 6$,

$$\pi^d = (12 - q_1^* - q_2^*)\, q_1^* - k = (12-6-3)\,3 - 8 = 1,$$

and $\pi^n$ denotes a firm's profit in the symmetric Nash equilibrium of the one-shot game,

$$\pi^n = (12 - q_1^* - q_2^*)\, q_1^* - k = (12-4-4)\,4 - 8 = 8.$$

So there is no incentive to deviate if

$$\begin{aligned} V^{eq} \geq V^{dev} &\Leftrightarrow \frac{\delta\pi^m}{1-\delta^2} \geq \pi^d + \frac{\delta\pi^n}{1-\delta} \\ &\Leftrightarrow \delta\pi^m \geq \left(1-\delta^2\right)\pi^d + (1+\delta)\,\delta\pi^n. \end{aligned}$$

Using the above values for $\pi^m$, $\pi^d$ and $\pi^n$, this condition simplifies to

$$28\delta \geq \left(1-\delta^2\right) + 8\,(1+\delta)\,\delta \Leftrightarrow f(\delta) \geq 0,$$

where

$$f(\delta) \equiv 28\delta - \left(1-\delta^2\right) - 8\,(1+\delta)\,\delta.$$

Note that we have $-1 = f(0) < 0 < f(1) = 12$ and

$$\begin{aligned} f'(\delta) &\equiv 28 + 2\delta - 8 - 16\delta \\ &= 14\,(1-\delta) + 6 > 0 \end{aligned}$$

for all $\delta < 1$. This means that there is a unique cut-off value $\delta_0 \in (0,1)$, defined by $f(\delta_0) = 0$, such that the firm that is supposed to produce nothing has no incentive deviate if, and only if, $\delta \geq \delta_0$.

What about situation (ii)? That is, what about the incentives to deviate for a firm that is supposed to produce $q_{i,t} = 6$? It may look as if such a firm should, if expecting the rival to choose $q_{j,t} = 0$, never have an incentive to deviate, because the firm would in the current period earn the monopoly profit, which cannot be made larger. However, this firm can, by deviating, improve on its profits in the *following* period (as well as all the future periods in which it is supposed to produce zero). We therefore need to investigate this case too. The firm can deviate in a way that lowers its current period profits with some arbitrarily small amount, by choosing a quantity that is slightly lower or slightly higher than $q_{i,t} = 6$. If doing that, the firm's profits would equal $\pi^d = 28 - \varepsilon$, where $\varepsilon$ is some positive number that can be made arbitrarily small. This action would also trigger the punishment phase, which means that the firm would earn the profit $\pi^n = 8$ in all the subsequent periods. Overall, the firm's present-discounted stream of profits if deviating in that way equals

$$V^{dev} = \pi^d + \frac{\delta\pi^n}{1-\delta} = 28 - \varepsilon + \frac{8\delta}{1-\delta}.$$

The firm's present-discounted stream of profits if not deviating equals

$$\begin{aligned} V^{eq} &= \pi^m + 0 + \delta^2\pi^m + 0 + \delta^4\pi^m + 0 + \cdots \\ &= \pi^m \left(1 + \delta^2 + \delta^4 + \delta^6 + \cdots\right) = \frac{\pi^m}{1-\delta^2} \\ &= \frac{28}{1-\delta^2}. \end{aligned}$$

So there is no incentive to deviate if

$$V^{eq} \geq V^{dev} \Leftrightarrow \frac{28}{1-\delta^2} \geq 28 - \varepsilon + \frac{8\delta}{1-\delta},$$

which holds for all $\varepsilon > 0$ if, and only if,

$$\frac{28}{1-\delta^2} \geq 28 + \frac{8\delta}{1-\delta}.$$

Simplifying this inequality yields

$$\begin{aligned} 28 \geq 28\left(1-\delta^2\right) + 8\delta\,(1+\delta) &\Leftrightarrow 28\delta \geq 8\,(1+\delta) \\ &\Leftrightarrow \delta \geq \frac{8}{20} = 0.4. \end{aligned}$$

That is, the firm that is supposed to produce $q_{i,t} = 6$ does not have an incentive to deviate if and only if

$\delta \geq 0.4$. Moreover, this condition is more stringent than the one required in situation (i) above: $\delta_0 < 0.4$. (This follows because $f'(\delta) > 0$ and $f(0.4) > 0$.)

Overall we can conclude that the specified strategies constitute a subgame perfect Nash equilibrium if and only if $\delta \geq 0.4$.

### Part (b)

*From the lecture slides:*

- The result that cooperation is possible for large enough values of $\delta$ is a special case of a more general result called the **Folk Theorem**.
- The **Folk Theorem**: In an infinitely repeated game with observable actions and in which the players are sufficiently patient:
  - Everything (that is feasible and individually rational) is an equilibrium.
- The Folk Theorem is, in a way, a problem for the theory:
  - What is the theory's prediction? If we can explain everything, then we cannot explain anything!
- The (pragmatic) approach taken by IO economists:
  - Assume the players can coordinate their behavior on some **"focal" equilibrium**.
  - For example, in a symmetric game, the players **coordinate on a symmetric equilibrium, and this equilibrium is Pareto efficient** from the point of view of these players (e.g., the firms).